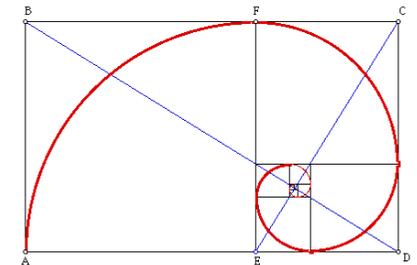
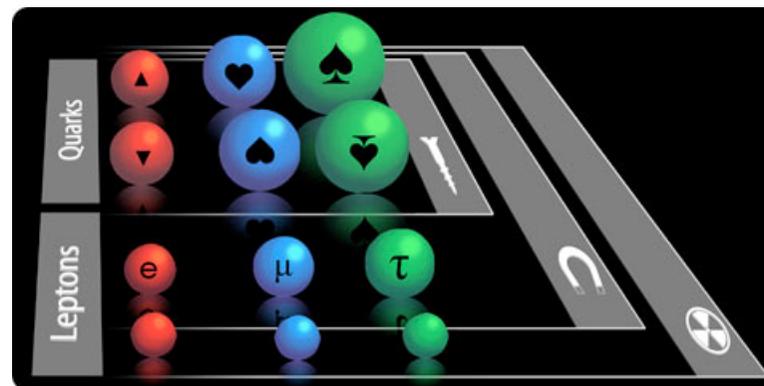
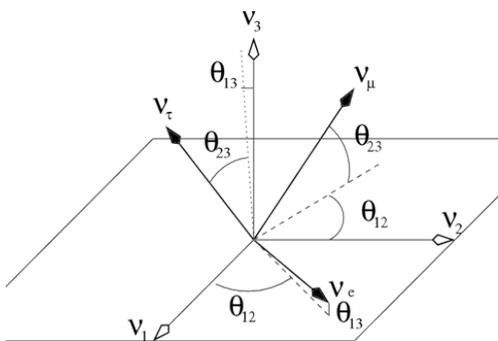


Neutrino Flavor Models

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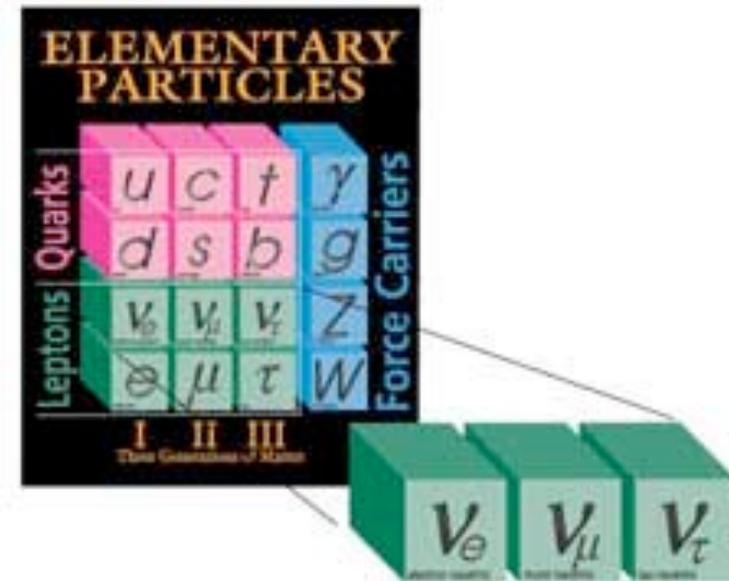
International Workshop for the Next Generation Nucleon Decay
and Neutrino Detector (NNNI5) and Unification Day 2 (UD2)



Discovery of Neutrino Oscillations:

$$\mathcal{P}_{\nu_\alpha \rightarrow \nu_\beta}(L) = \sum_{ij} U_{i\alpha} U_{i\beta}^* U_{j\alpha}^* U_{j\beta} e^{-\frac{i\Delta m_{ij}^2 L}{2E}}$$

surprises, confusion, excitement
for beyond SM physics theory!



3 Neutrino “Reference” Picture:

data (w/exceptions*) consistent with 3ν mixing picture

intriguing pattern of masses, mixings:
paradigm shift for SM flavor puzzle

Of course — the picture may not be this “simple”!

Many Questions Remain

- **How many light neutrinos?**

Anomalies: LSND, MiniBooNE, Gallium, Reactor
eV-scale sterile neutrinos? But tension still with all oscillation data

For now restrict to 3-family neutrino models only

SM  ν SM

- **Still, many questions:**

Nature of neutrino mass suppression? Majorana? Dirac?

Mass hierarchy? **Lepton mixing angle pattern?** CP violation?

Implications for BSM paradigms? Connections to other NP?

The Data: Fermion Masses

Quarks,
charged leptons:
hierarchy!

$$m_u \simeq 1 - 4 \text{ MeV}$$

$$m_d \simeq 4 - 8 \text{ MeV}$$

$$m_e = 0.511 \text{ MeV}$$

$$m_c \simeq 1 \text{ GeV}$$

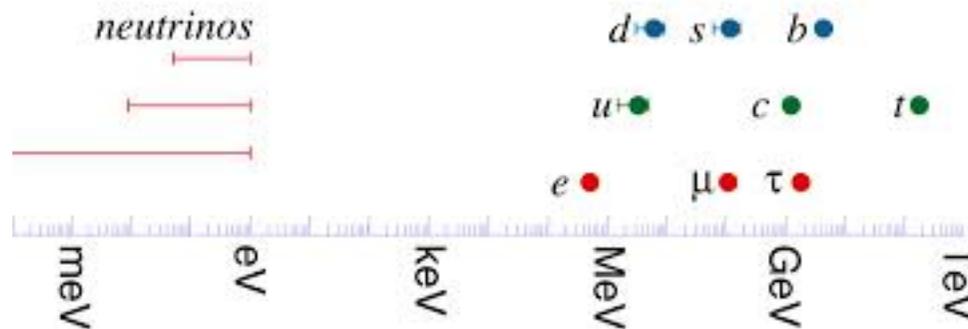
$$m_s \simeq 100 \text{ MeV}$$

$$m_\mu = 105 \text{ MeV}$$

$$m_t \simeq 175 \text{ GeV}$$

$$m_b \simeq 5 \text{ GeV}$$

$$m_\tau \simeq 1.8 \text{ GeV}$$

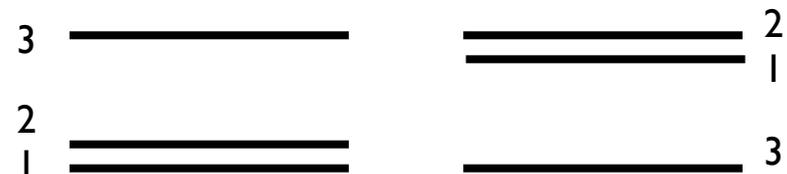


Neutrinos: $\Delta m_{ij}^2 \equiv m_i^2 - m_j^2$

$$\Delta m_{\odot}^2 = |\Delta m_{12}^2| \simeq 8 \times 10^{-5} \text{ eV}^2$$

$$\Delta m_{\oplus}^2 = |\Delta m_{23}^2| \simeq 2 \times 10^{-3} \text{ eV}^2$$

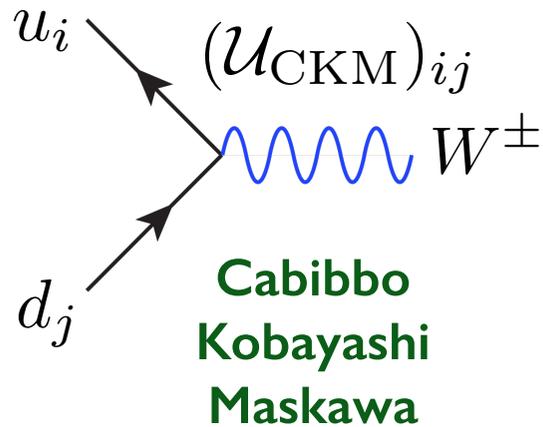
Normal Hierarchy (NH) **Inverted Hierarchy (IH)**



individual masses: only limits from direct searches, cosmology

The Data: Fermion Mixing rotation matrices

Quarks:



$$U_{\text{CKM}} = \mathcal{R}_1(\theta_{23}^{\text{CKM}}) \mathcal{R}_2(\theta_{13}^{\text{CKM}}, \delta_{\text{CKM}}) \mathcal{R}_3(\theta_{12}^{\text{CKM}})$$

$$\theta_{12}^{\text{CKM}} = 13.0^\circ \pm 0.1^\circ = \theta_C \quad \text{(Cabibbo angle)}$$

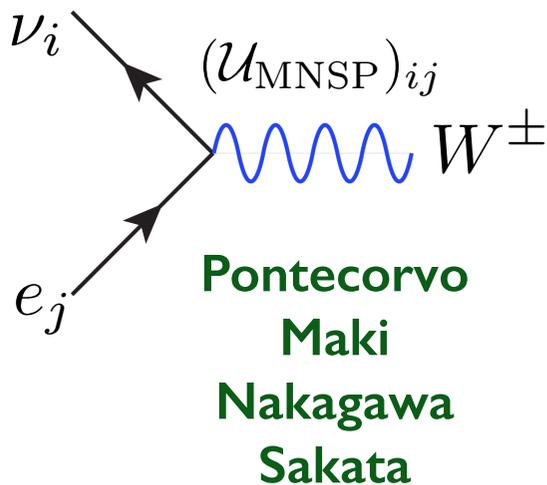
$$\theta_{23}^{\text{CKM}} = 2.4^\circ \pm 0.1^\circ$$

$$\theta_{13}^{\text{CKM}} = 0.2^\circ \pm 0.1^\circ$$

$$\delta_{\text{CKM}} = 60^\circ \pm 14^\circ$$

**3 “small” angles
1 large CP phase**

Leptons:



$$U_{\text{MNSP}} = \mathcal{R}_1(\theta_{23}) \mathcal{R}_2(\theta_{13}, \delta_{\text{MNSP}}) \mathcal{R}_3(\theta_{12}) \mathcal{P}$$

NH; best fit +/- 1 sig (3sig range)

$$\theta_{12} = 33.48^\circ_{-0.75^\circ}^{+0.78^\circ} \quad (31.29^\circ - 35.91^\circ)$$

$$\theta_{23} = 42.3^\circ_{-1.6^\circ}^{+3.0^\circ} \quad (38.2^\circ - 53.3^\circ)$$

$$\theta_{13} = 8.50^\circ_{-0.21^\circ}^{+0.20^\circ} \quad (7.85^\circ - 9.10^\circ)$$

phases (if Majorana)

**2 large angles,
1 “small” angle**

**fits from Gonzalez-Garcia et al. '14,
see also Forero et. al '14, Capozzi et al. '13**

(Broad) Theoretical Implications

SM probes: only sensitive to subset of model parameters

Can only provide classes of viable models!
(need NP/precision measurements to distinguish)

Basic categories:

Majorana or Dirac neutrinos?

most important
question (by far)



mechanism for suppressing neutrino mass scale

lepton mixing angles: symmetry or anarchy?

We're still in the very early stages of mapping out the
theory space of acceptable models!

Mass Generation

Quarks, Charged Leptons

“natural” mass scale tied to electroweak scale
Dirac mass terms, parametrized by Yukawa couplings



$$Y_{ij} H \cdot \bar{\psi}_{Li} \psi_{Rj}$$

top quark: $O(1)$ Yukawa coupling
rest: suppression (flavor symmetry)

Neutrinos beyond physics of Yukawa couplings!

Options: **Dirac**



Majorana



Majorana first:



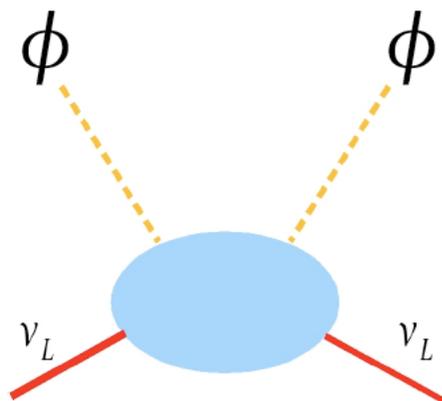
advantages: naturalness, leptogenesis, $0\nu\beta\beta$

SM at NR level: Weinberg dim 5 operator

$$\frac{\lambda_{ij}}{\Lambda} L_i H L_j H$$

(if $\lambda \sim O(1)$ $\Lambda \gg m \sim O(100 \text{ GeV})$ but wide range possible)

Underlying mechanism: examples



Type I seesaw ν_R (fermion singlet)

Type II seesaw Δ (scalar triplet)

Type III seesaw Σ (fermion triplet)

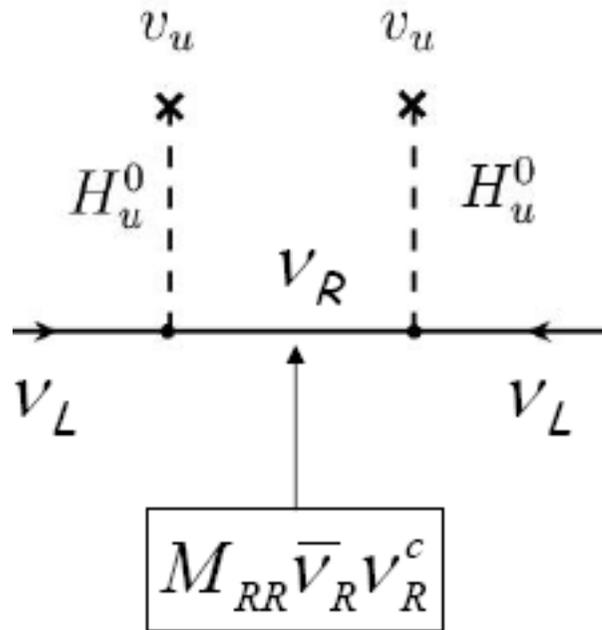
+ variations

Prototype: Type I seesaw

Minkowski; Yanagida;
Gell-Mann, Ramond, Slansky;
Mohapatra, Senjanovic;...

right-handed neutrinos:

$$Y_{ij} L_i \nu_{Rj} H + M_{Rij} \nu_{Ri} \nu_{Rj}^c$$



$$\mathcal{M}_\nu = \begin{pmatrix} 0 & m \\ m & M \end{pmatrix} \quad \begin{array}{l} m \sim \mathcal{O}(100 \text{ GeV}) \\ M \gg m \end{array}$$

$$m_1 \sim \frac{m^2}{M} \quad m_2 \sim M \gg m_1$$

$$\nu_{1,2} \sim \nu_{L,R} + \frac{m}{M} \nu_{R,L}$$

advantages: naturalness, connection to grand unification, leptogenesis,...

disadvantage: testability (even at low scales)

Different in Type II, III: new EW charged states — visible at LHC?

Many other ideas for Majorana neutrino masses...



more seesaws (double, inverse,...),
loop-induced masses (Babu-Zee, ...),
SUSY with R-parity violation, RS models,
higher-dimensional (>5) operators,...

What about Dirac masses?

Less intuitive, but suppression mechanisms exist...



extra dimensions, extra gauge symms (non-singlet ν_R), SUSY breaking,...

General themes:

Trade-off b/w naturalness and testability.

Much richer than quark and charged lepton sectors.

Lepton (and Quark) Mixing Angle Generation

Standard paradigm: **spontaneously broken flavor symmetry**

$$Y_{ij} H \cdot \bar{\psi}_{Li} \psi_{Rj} \longrightarrow \left(\frac{\varphi}{M} \right)^{n_{ij}} H \cdot \bar{\psi}_{Li} \psi_{Rj} \quad \text{Froggatt, Nielsen}$$

Quarks:

hierarchical masses, small mixings: **continuous** family symmetries

CKM matrix: small angles and/or alignment of left-handed mixings

$$\mathcal{U}_{\text{CKM}} = \mathcal{U}_u \mathcal{U}_d^\dagger \sim 1 + \mathcal{O}(\lambda) \quad \lambda \sim \frac{\varphi}{M}$$

Wolfenstein parametrization: $\lambda \equiv \sin \theta_c = 0.22$

suggests Cabibbo angle (or some power) as a flavor expansion parameter

The Flavor Puzzle, Rejuvenated

Flavor puzzle of SM is notoriously difficult...

Still difficult in ν SM, but more interesting --

One primary reason: two large mixing angles!

3-family models: handwave a bit (in diagonal charged lepton basis)

3	small angles	→	~ diagonal \mathcal{M}_ν	} (“easy”)
1	large, 2 small	→	~ Rank $\mathcal{M}_\nu < 3$	
3	large angles	→	anarchical \mathcal{M}_ν	} (“harder”)
2	large, 1 small	→	fine-tuning, non-Abelian	

Anarchy vs. Structure

→ The question: is θ_{13} large or small?

$$\theta_{13} \simeq 9^\circ \pm 1^\circ$$

New case for anarchy: de Gouvea and Murayama,...

Focus here on structure (symmetry):

Paradigm: discrete non-Abelian family symmetry

(e.g. some subgroup of $SO(3)$ or $SU(3)$, broken to some appropriate coset space)

One issue/challenge: many theoretical starting points

Role of Small (Cabibbo-sized) Corrections

Quark sector:

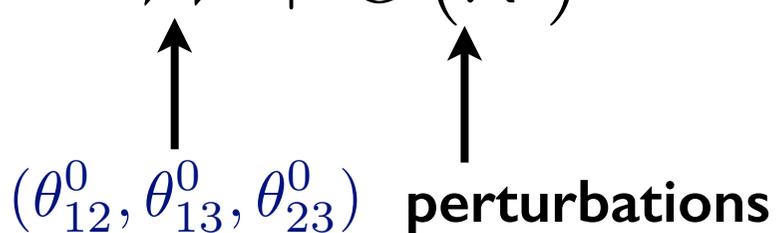
$$\mathcal{U}_{\text{CKM}} \sim 1 + O(\lambda_C)$$

Cabibbo angle λ_C (or some power) as a flavor expansion parameter

Lepton sector:

$$\mathcal{U}_{\text{MNSP}} \sim \mathcal{W} + O(\lambda')$$

$(\theta_{12}^0, \theta_{13}^0, \theta_{23}^0)$ perturbations



choice of “bare” mixing angles

Unification paradigm (broad sense): useful to take

$$\lambda' = \lambda_C$$

ideas of quark-lepton complementarity and “Cabibbo haze”

Raidal '04, Minakata+Smirnov, many others...
 (“haze” terminology from Datta, L.E., Ramond '05)

Long before measurement, conjecture that θ_{13} is a Cabibbo effect

$$\theta_{13} \sim \frac{\lambda_C}{\sqrt{2}} \sim \lambda_C \cos \theta_{23}^0 \quad \text{Ramond, others...}$$

(general idea often called “charged lepton corrections”) $\mathcal{U}_{\text{MNSP}} \sim \mathcal{U}_{\text{CKM}}^\dagger \mathcal{W}$

good fit to data, but nontrivial to implement...

one reason: now $\sim \lambda_C$ corrections floating around

The Flavor Puzzle in the ν SM

Pre-Reactor Meas. most models: $\theta_{23}^0 = 45^\circ$ $\theta_{13}^0 = 0^\circ$

Choices for “bare” solar angle θ_{12}^0 :

(i) within $\sim \lambda_C^2$ of exp:

tri-bimaximal mixing

“the beautiful matrix
with the ugly name”

$$\tan \theta_{12}^0 = \frac{1}{\sqrt{2}} \quad \theta_{12}^0 = 35.26^\circ$$

Harrison, Perkins, Scott

(100s of papers...some key players
here at this meeting!)

others, such as **golden ratio** mixing $\phi = (1 + \sqrt{5})/2$

$$\tan \theta_{12} = \phi^{-1} \quad \theta_{12} = 31.72^\circ \quad \text{or} \quad \cos \theta_{12} = \frac{\phi}{2} \quad \theta_{12} = 36^\circ$$

Ramond; Kajiyama et al.;

LE+Stuart (+Ding); Feruglio et al.,...

Rodejohann et al.,...

(ii) within $\sim \lambda_C$ of exp: **QLC** idea of Raidal, Minakata/Smirnov,...

bimaximal mixing $\tan \theta_{12}^0 = 1$

“Top-down” approach: detailed model-building

example: tri-bimaximal (TBM/HPS) mixing

$$U_{\text{MNSP}}^{(\text{HPS})} = \begin{pmatrix} \sqrt{\frac{2}{3}} & -\frac{1}{\sqrt{3}} & 0 \\ \frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & -\frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{2}} \end{pmatrix} \quad (\sim \text{Clebsch-Gordan coeffs!})$$

Meshkov; Zee,...

Obtained w/in many discrete non-Abelian subgroups of $SO(3)$, $SU(3)$

$$\mathcal{G}_F = \mathcal{A}_4, \mathcal{S}_4, \mathcal{T}', \Delta(3n^2), \dots$$

Many papers!!
many key players...

(note: details encoded in rather complicated flavon sector)

“Bottom-up” approach: residual symmetries

$$\mathcal{G}_F \xrightarrow{\text{red arrow}} \mathcal{G}_e \times \mathcal{G}_\nu \quad \text{Lam; Ding et al.,...}$$

pure group theory argument: e.g. “minimal” group S_4 for TBM

Post-Reactor Meas.

“Top-down” approach:

(1) Keep $\theta_{23}^0 = 45^\circ$ $\theta_{13}^0 = 0^\circ$

(i) within $\sim \lambda_C^2$ of exp: need to **control corrections**

TBM (or other mixing scenarios) as leading order framework

many papers...

(ii) within $\sim \lambda_C$ of exp: resurgence?

(2) Modify $\theta_{23}^0 = 45^\circ$ $\theta_{13}^0 = 0^\circ$

θ_{13} numerology? drop maximal θ_{23} ?

“Bottom-up” approach:

larger groups? implications for CP violation?

Holthausen et al., King et al., Hagedorn et al., many others,...

CP Violation

Model-building: **spontaneous** v. explicit CP violation

Generalized CP transformations:

CP tmns as automorphisms **Grimus, Rebelo '90's**

for discrete groups: **Holthausen et al., Chen et al.,...**

family symmetry: $\phi \rightarrow \rho(g)\phi$

generalized CP: $\phi \rightarrow U\phi^*$ (**not** $\phi \rightarrow \phi^*$)

consistency condition  $U\rho(g)^*U^{-1} = \rho(g')$

Moral: CP and family symmetries can be inextricably intertwined

Lots of recent work along these lines... **see e.g. Ding et al.; Girardi et al.; L.E., Garon, Stuart; many others...**

eV-scale Sterile Neutrinos?

If eV-scale sterile ν present, many implications:

mass hierarchies? $0\nu\beta\beta$ GUT connections?

mixing pattern and residual symmetries? CP?

example: $n_s = 1$ $\theta_{14} \sim \theta_{13}$ same origin?

Back to the drawing board!!

Conclusions and Outlook

The SM flavor puzzle is a difficult but intriguing problem —
we're just beginning to scratch its surface!

Most important question: **Majorana** or **Dirac** neutrinos?

New insights/approaches from the lepton data

Naturalness/testability tradeoff

Precision measurements/NP needed to distinguish models

Lots of ideas, lots of room for more

Stay tuned!